

## **How Much Should We Value Uncertainty in Energy System Planning?**

J.F. DeCarolis, K. Hunter

Department of Civil, Construction, and Environmental Engineering  
North Carolina State University Campus Box 7908  
Raleigh, NC, USA 27695

### **Abstract**

Energy system models should reflect the reality that planners must make decisions prior to the resolution of large future uncertainties. Multi-stage stochastic optimization, which embeds the probability of different outcomes within an event tree, optimizes over all possible outcomes to yield a single, near-term decision strategy. Given the computational difficulty; however, stochastic optimization applied to energy system models has typically been done as a two-stage formulation. The Temoa model has been designed to operate in a high performance computing (HPC) environment and can be used to conduct stochastic optimization over multiple uncertain time stages. However, metrics are required to quantify how the number of uncertain stages, choice of discount rate, and branches per node within the event tree affect the cost of uncertainty and the value of the near term hedging strategy. This paper employs both the expected value of perfect information (EVPI) and the expected cost of ignoring uncertainty (ECIU) as metrics to quantify the value of performing stochastic optimization.

### **1. Introduction**

A key challenge associated with the application of energy models is accounting for future uncertainty. Most model-based analyses assume a set of exogenous scenario-based assumptions to capture potential outcomes. Each individual scenario; however, assumes perfect foresight. While the resultant family of scenarios is meant to capture the range of potential future outcomes, they are of limited value to decision makers who must make a single set of near-term decisions before uncertainty is resolved (Morgan and Keith, 2008). To address future uncertainty within the model formulation, stochastic optimization can be applied by building an event tree and optimizing over all possible future outcomes, each weighted by a subjective probability of occurrence (Loulou et al., 2004). The result is a stochastic solution, which represents a near-term hedging strategy that accounts for potential future outcomes and puts the decision maker in a position to take recourse action as the uncertainty is resolved.

Compared to any single perfect foresight scenario, the stochastic solution is necessarily more expensive because it simultaneously accounts for all specified scenario outcomes. A critical issue is whether the resultant hedging strategy is worth the extra cost. There are two relevant cost metrics to assess the efficacy of the hedging strategy. The first metric is the expected cost of perfect information (EVPI), which is the difference in cost between the stochastic solution and the expected value of the perfect foresight scenarios (Morgan and Henrion, 1990; Clemen and Reilly, 2004). The EVPI represents how much the decision makers would be willing to pay in order to eliminate the uncertainty.

In the real world; however, decision makers often cannot pay to resolve uncertainty, as the uncertainty itself is beyond their control. Rather, decision makers wish to know how much money the hedging strategy saves relative to the expected cost when uncertainty is ignored. The expected cost of ignoring uncertainty (ECIU) can be used to value the stochastic solution relative to planning that ignores future uncertainty and may require more drastic recourse action (Birge and Louveaux, 1997; van der Weijde and Hobbs, 2012).

Both the EVPI and ECIU will vary depending on the parameterization of the energy system model and stochastic formulation via the event tree. Because energy systems have long-lived infrastructure, significant turnover in capital stock takes place over a relatively long period of time. As a result, the application of stochastic optimization over multiple time stages (i.e.,  $>2$ ) can yield valuable, planning-relevant insight. While stochastic optimization has been performed on energy system models in the past (e.g., Kanudia et al., 1998; Loulou and Kanudia, 1999; Bosetti and Tavoni, 2009; Babonneau et al., 2012), Temoa is unique because it was designed to operate in a high performance computing (HPC) environment and can therefore solve larger event trees (Hunter et al., 2012). While Temoa extends our capability to do uncertainty analysis, the curse of dimensionality means that problem formulation can still quickly overwhelm the available compute hardware.

The purpose of this paper is to exercise a simple energy system representation in Temoa to see how the EVPI and ECIU values scale as a function of discount rate and the number of uncertain stages. We hypothesize that as the discount rate and/or the number of uncertain time stages increase, the EVPI and ECIU will converge to a limit. The results of this study can be

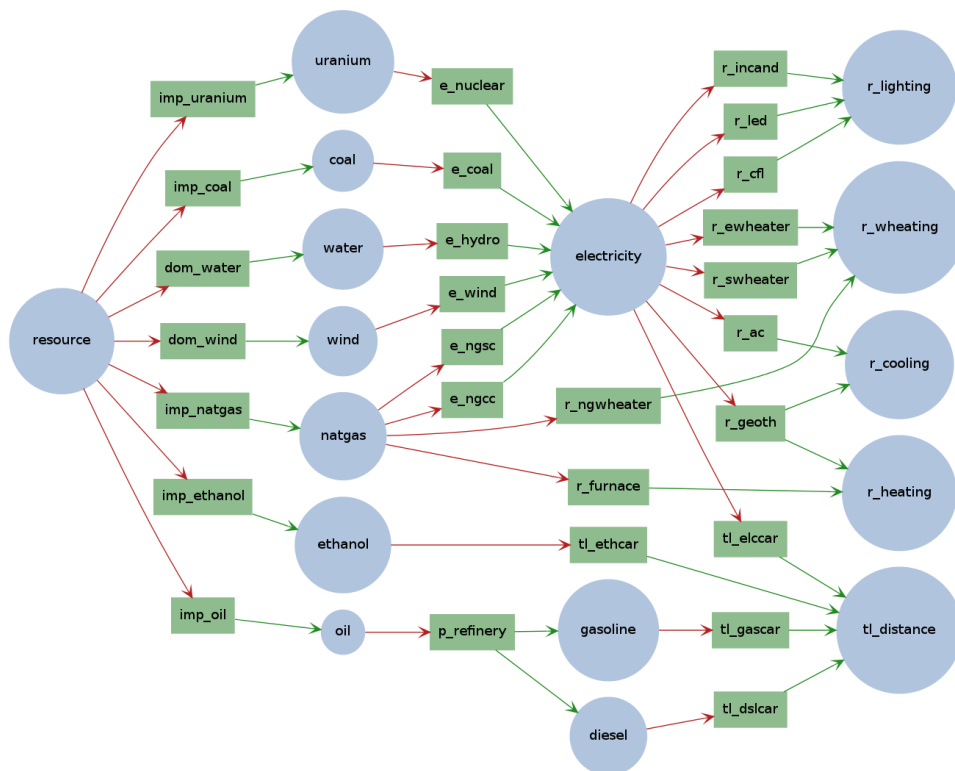
used to suggest practical limits on the necessary size of a stochastic energy system model contingent on the chosen discount rate.

## 2. Methods

The EVPI and ECUI are calculated for a simple test system called ‘Temoa Island’ as a function of the global discount rate and number of anticipative time stages (i.e., time stages in which uncertainty is resolved). We first outline the simple energy system representation followed by a more detailed description of the EVPI and ECUI calculations.

### 2.1 Temoa\_Island: a simple energy system representation

We have developed a simple test system used for Temoa verification exercises, which we refer to as ‘Temoa\_Island.’ The system map is presented below in Figure 1. For simplicity, Temoa\_Island contains no pre-existing capacity.



**Figure 1.** Representation of Temoa\_Island, a simple energy system created for testing purposes. Commodities are represented as blue circles, and processes are represented as green boxes. The abbreviation ‘imp’ indicates import, ‘dom’ indicates a domestic resource, ‘e’ represents an electric

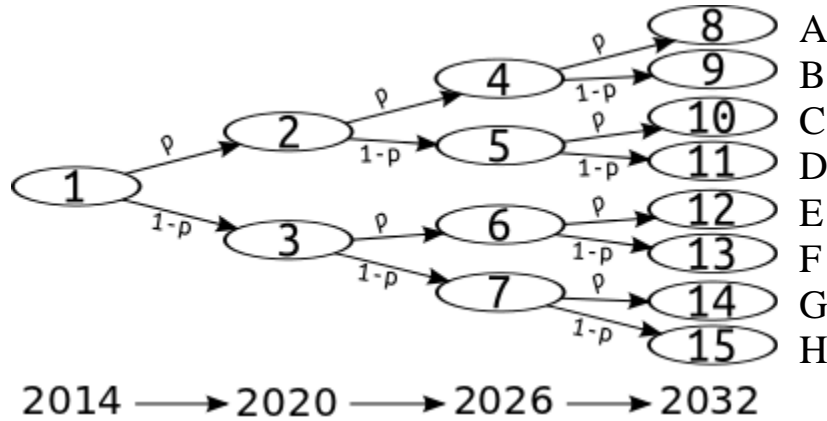
generator, and 'p' represents a process. Select demand technologies and end-use demands are represented in the residential ('r') and light duty transportation ('tl') sectors. In addition, 'ngcc' refers to natural gas combine-cycle, and 'ngsc' to natural gas simple-cycle.

The time horizon spans 2014 to 2032 and consists of 4 model time periods each spanning 6 years. Intra-annual demand variability is represented by 6 annual time slices (the number in parentheses represents fraction of the year): summer-day (0.125), summer-night (0.125), winter-day (0.125), winter-night (0.125), intermediate-day (0.25), and intermediate-night (0.25). Annual growth in residential lighting, space heating and cooling, and water heating demand is 0.86%, which is based on the annual growth in U.S. residential primary energy demand from 2001 – 2010 (EIA, 2011). Likewise, annual growth in light duty transportation demand of 0.002% is based on the growth rate in U.S. transportation petroleum demand from 2001 – 2010 (EIA, 2011). The estimated commodity prices as well as technology cost and efficiency estimates are derived from U.S. data sources: Commodity prices are drawn from EIA (2012), energy generation data is taken from EIA (2010), and demand technology from Shay et al. (2008).

## **2.2 Stochastic formulation**

In this simple application, the stochastic parameter is a period-by-period upper bound on CO<sub>2</sub> emissions. We assume that at the beginning of every model time period (every six years) there is a parliamentary election, and the political party in power sets the course of action. If the green party is in the majority, then they implement a constraint to reduce CO<sub>2</sub> emissions by 2.3% annually. If, on the other hand, the pro-business party gets elected directly following a green party majority, CO<sub>2</sub> emissions are allowed to grow at 2.3%. If the pro-business party gets elected for a second time in a row, then the constraint on CO<sub>2</sub> emissions is lifted entirely for that time period. As a result, there are two branches per node and 3 anticipative time stages for a total of 8 scenarios.

The only non-anticipative stage (i.e., with no uncertainty revealed) is 2014. Figure 2 illustrates the event tree with 3 anticipative stages (2020, 2026, and 2032).



**Figure 2.** Sample event tree with one non-anticipative stage in 2014 followed by three stages in which uncertainty about a period-specific CO<sub>2</sub> bound is revealed. Nodes in the event tree are numbered starting from the left and moving down and to the right. The sequence 1→2→4→8 (Scenario A) represents no CO<sub>2</sub> policy throughout, while the sequence 1→3→7→15 (Scenario H) represents a CO<sub>2</sub> policy throughout.

Once the stochastic solution (i.e., hedging strategy) is obtained, the EVPI can be calculated as follows:

$$EVPI = C_{hedge} - \sum_{s \in S} p_s \cdot C_s$$

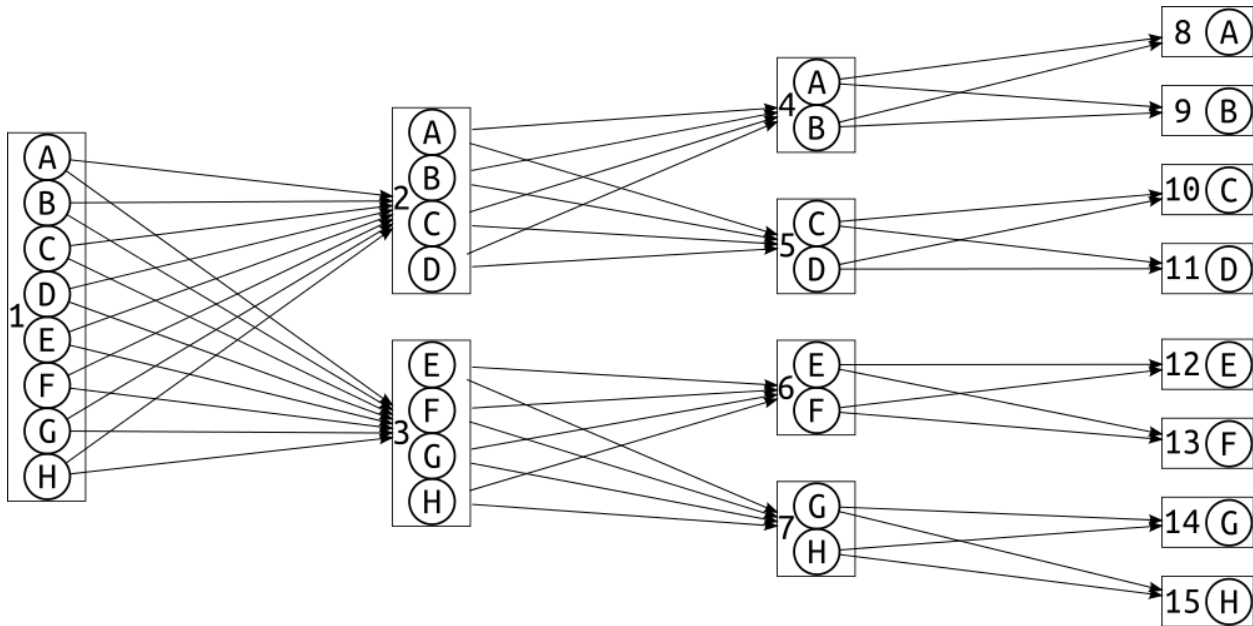
where  $C_{hedge}$  is the cost of the hedging strategy and the second term represents the expected value of all the perfect foresight scenarios, which is given by the sum product of the scenario probabilities ( $p_s$ ) and the scenario-specific costs ( $C_s$ ). Note that each perfect foresight scenario corresponds to a unique path through the event tree.

The ECIU provides a valuation of the stochastic solution compared to ignoring uncertainty by assuming an initial perfect foresight scenario and taking recourse actions as uncertainty is resolved. Suppose we only consider two time stages (i.e., Nodes 1-3 in Figure 1 above). If we further suppose that there will be no CO<sub>2</sub> limit in 2020, we solve for the optimal decision variable values associated with Nodes 1 and 2. Once this decision is made, the Node 1 variables in 2014 assuming no CO<sub>2</sub> policy are now fixed. But what if we are wrong, and there is a CO<sub>2</sub> policy in 2020? To estimate the recourse cost in this case, the Stage 1 decision variables are fixed and we solve for the optimal Stage 2 variables at Node 3 with the CO<sub>2</sub> policy. The resultant ECIU in the 2-stage case, with an initial assumption of no CO<sub>2</sub> policy, is given by:

$$ECIU_2 = (C_{1|2} + p \cdot C_{2|2} + (1 - p) \cdot C_{3|2}) - C_{hedge}$$

where  $C_{1|2}$  represents the cost at Node 1 when the outcome is assumed to be Node 2,  $C_{2|2}$  represents the Node 2 cost if Node 2 is the actual outcome, and  $C_{3|2}$  represents the Node 3 cost if Node 3 is the actual outcome instead of the assumed Node 2. Note that if Node 2 is the actual outcome, no recourse action is required because we guessed correctly; however, if Node 3 is the outcome then potentially expensive recourse action is required. Note that we get a single expected value for ECIU for the scenario in which we assume the outcome is Node 2 in 2020. The ECIU calculation would need to be repeated for the scenario where Node 3 is the expected outcome in 2020. More generally, there are as many ECIU estimates as there are perfect foresight scenarios in the event tree.

When solving event trees with multiple uncertain stages, the ECIU calculation is a bit more complex. The optimal decision variable values (assuming a given perfect foresight scenario) must be fixed in successive time stages and the model re-optimized to find the cost of recourse associated with each uncertain stage. We illustrate the procedure with another example. Suppose, as in Figure 1, there are 3 uncertain stages. Further suppose we guessed that there would be no CO<sub>2</sub> policy through the model time horizon because the pro-business party builds a permanent majority (Scenario A). However, in 2020 the green party wins a majority in parliament and institutes a CO<sub>2</sub> policy. The model must take recourse action in 2020 – but because it assumes perfect foresight through the end of the time horizon in 2032—we have to assume a specific scenario (E-H) in order to optimize the decision variables associated with Node 3 in 2020. In other words, there are 4 possible recourse options associated with Node 3 based on which scenario is chosen through 2032. This is graphically illustrated in Figure 3 below.

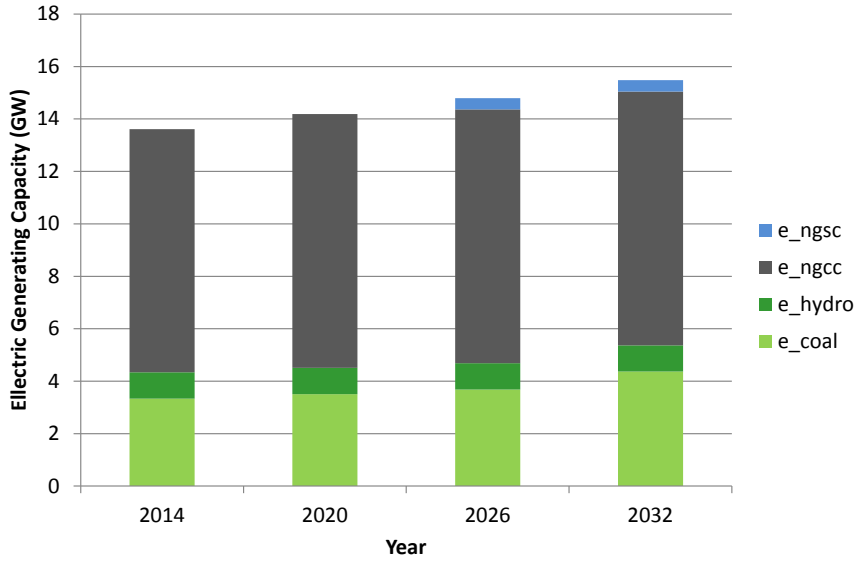


**Figure 3.** Feasible scenarios as a function of model time stage associated with an event tree containing 3 uncertain stages. Each numbered box represents the same numbered node as in Figure 2, but each circled letter represents the assumption at that node of the final scenario that will occur. When optimizing Node 2, for example, there are 4 sets of solutions based on which perfect foresight scenario from Node 2 is assumed through 2032.

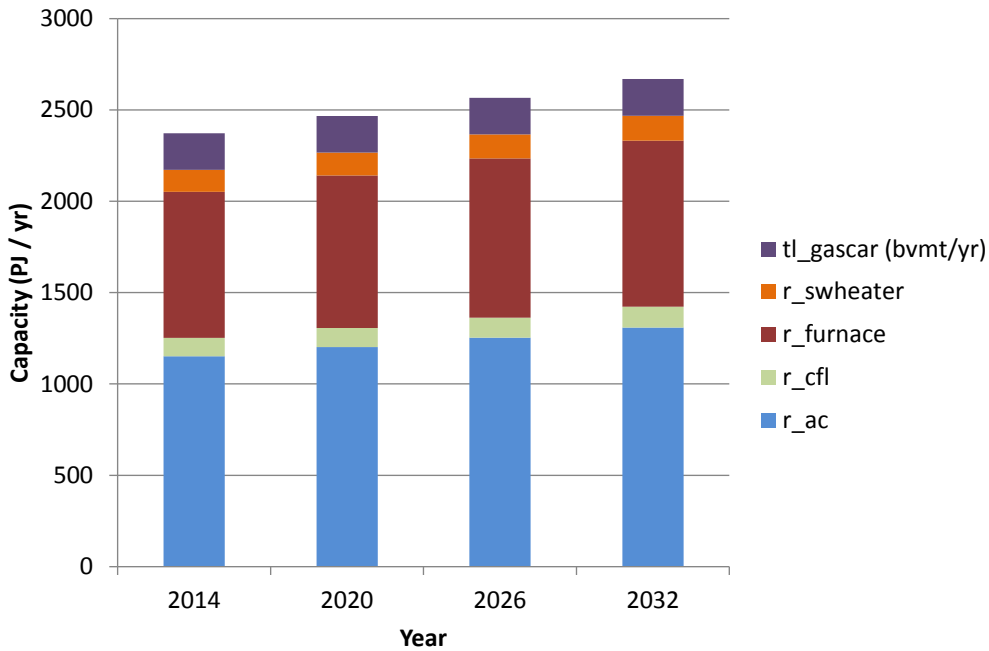
Because the recourse action depends in part on the assumption of perfect foresight scenario through the end of the model time horizon, the model must be solved recursively at each node based on the number of remaining feasible scenarios. The calculation of ECIU is computationally intensive and ideally suited for a high performance computing environment. In the simple 4 stage, 8 scenario example illustrated in Figure 3, there are 64 paths through the event tree that begin with initially assuming Scenario A in 2014. Since there are a total of 8 perfect foresight scenarios that can be assumed in 2014, the total number of individual scenario runs necessary to calculate the ECIU is 840. The model runs were run in parallel on Cygnus, a compute cluster consisting of 11 nodes and 88 cores.

### 3. Results

Before proceeding to the results from the stochastic optimization, the base case results with no CO<sub>2</sub> policy and a 5% global discount rate are provided below for reference. Figure 4 presents the results from the electric sector, and Figure 5 presents results from the end-use sectors.



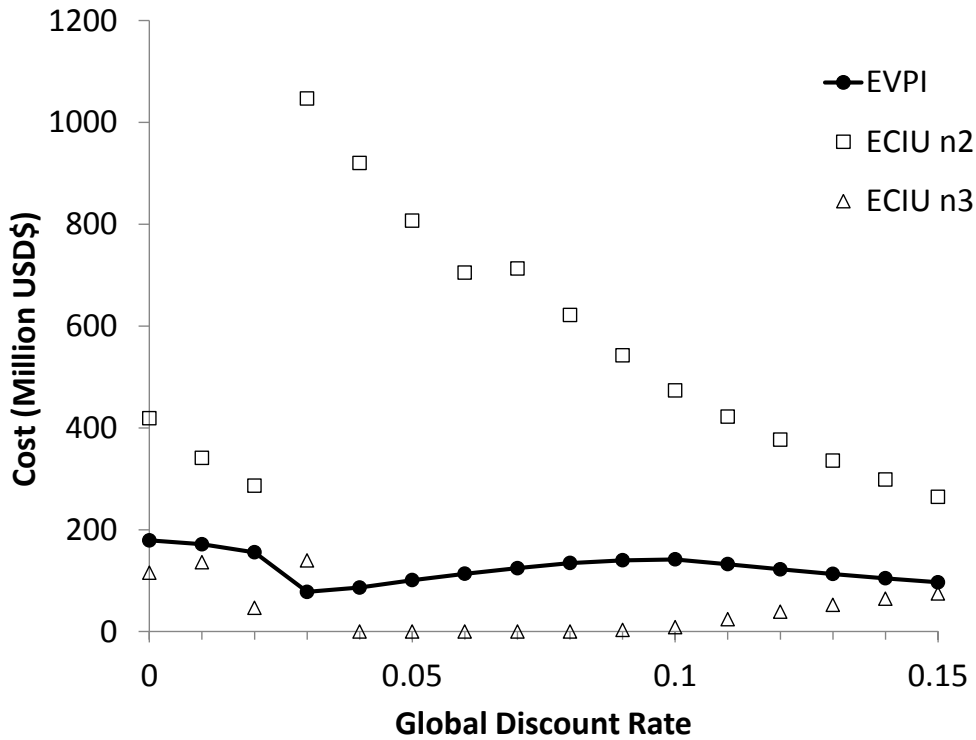
**Figure 4.** Installed electric generation capacity in the Temoa\_Island base case. There is no pre-existing capacity in the system, and e\_hydro is limited to 1 GW each model time period.



**Figure 5.** Installed demand technologies in the Temoa\_Island base case. Note that the output units for all demand technologies are PJ/yr, with the exception of light duty transport, which is in billions of vehicle-miles traveled/yr.

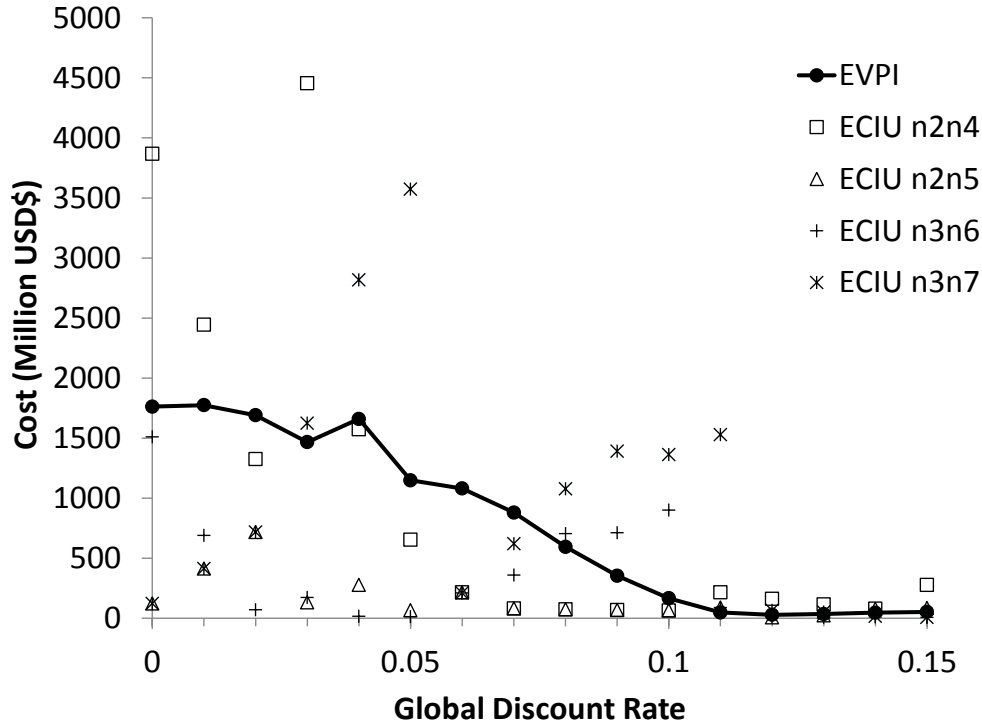


The number of time stages in the stochastic model was varied from 2 to 4. For each time stage version of the model, the global discount rate was varied from 0-15% in 1% increments. The 2-stage case is presented below in Figure 6.



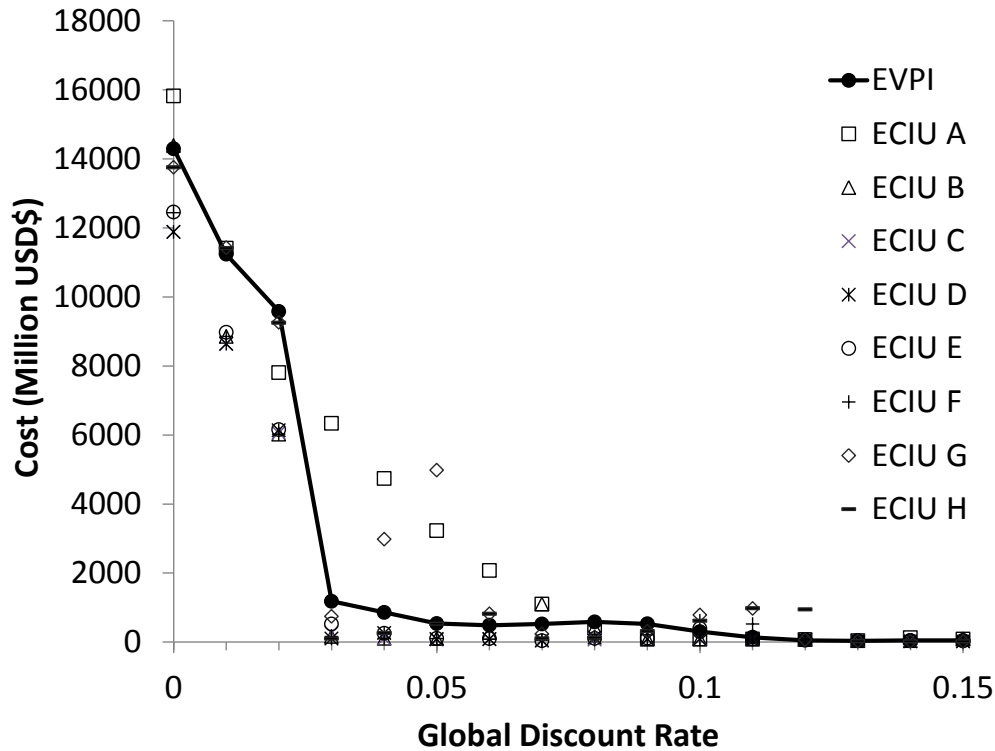
**Figure 6.** EVPI and ECIU in the 2-stage case as a function of the global discount rate. The designation following the ECIU indicates the node corresponding to the anticipated outcome (e.g., ‘n2’=Node 2, which corresponds to no CO<sub>2</sub> policy in 2020).

Figure 6 indicates that neither the EVPI nor ECIU change monotonically as a function of discount rate. In this simple 2-stage model, variations in the discount rate lead to changes in investment patterns that can significantly alter the required recourse actions. In addition, the short time horizon of 12 years results in a lower system cost, which suggests that even minor variations in installed capacity can produce large economic effects. For example, the ECIU associated with anticipating no CO<sub>2</sub> policy (‘ECIU n2’) increases dramatically when moving from a 2% to 3% discount rate. The results from the 3-stage case are presented below in Figure 7.



**Figure 7.** EVPI and ECIU in the 3-stage case as a function of the global discount rate. The designation following the ECIU indicates the nodes corresponding to the anticipated outcome (e.g., ‘n2n5’=Node 2,5 which corresponds to no CO<sub>2</sub> policy in 2020 followed by a CO<sub>2</sub> policy in 2026).

Compared to the 2-stage results, the EVPI and ECIU from the 3-stage case show greater predictability as a function of discount rate. The EVPI declines nearly monotonically with an increasing discount rate. Not surprisingly, the highest ECIU values are associated with the 2 extreme scenarios, the CO<sub>2</sub> policy (‘ECIU n3n7’) or no CO<sub>2</sub> policy (‘ECIU n2n4’) for the entire time horizon. Discount rates higher than 10% result in both EVPIs and ECIUs that are low and show lower relative variability compared to that observed at lower discount rates. The results from the 4-stage case are presented in Figure 8.



**Figure 8.** EVPI and ECIU in the 4-stage case as a function of the global discount rate. The designation following the ECIU indicates the scenario from Figure 1 anticipated in 2014.

When extending the model from 3 to 4 stages, the values of the EVPI and ECIU increase significantly at lower discount rates. Given the larger event tree in the 4-stage case, the relative weight of any single branch is less, and as a result, the expected outcomes become more predictable. Unlike in previous cases, the EVPI declines monotonically as the discount rate increases, and the ECIU estimates show less relative variability compared to the models with fewer time stages.

#### 4. Discussion

When performing stochastic optimization to inform energy system planning, it is useful to have metrics that allow modelers to quantify the cost of addressing uncertainty. The EVPI indicates a decision maker's willingness to pay to resolve future uncertainty. However, the ability to eliminate the uncertainty often does not exist in any practical sense. In such cases, the stochastic

solution provides a hedge against different future outcomes, and we can value the hedging strategy by calculating the ECIU. In this paper, we have applied the EVPI and ECIU as metrics in a simple energy system representation in which the discount rate and number of time stages were varied under an uncertain CO<sub>2</sub> control regime. As expected, the EVPI and ECIU converge to lower limits as the discount rate increases. Interestingly; both the EVPI and ECIU exhibit higher proportional variability with fewer time stages, in part because the cost of recourse can have a large impact in systems with lower total costs, resulting from a narrow time horizon.

Computing these metrics, in particular the ECIU, are computationally intensive. To begin, 48 multistage stochastic formulations (2, 3, or 4 stages each tested at 16 discount rates) were solved. At a given discount rate, the ECIU calculations required 6 runs in the 2-stage case, 52 runs in the 3-stage case, and 840 runs in the 4-stage case. Since 16 discount rates were tested at each stage number, the total ECIU runs required were 14,368. The EVPI calculations were based on runs completed for the ECIU calculations. The total required runs, both serial and stochastic, were 14,461. This modeling exercise could only be conducted in an HPC environment. Given the required computational effort, analyses in the near-term will likely be limited to a maximum of 6-7 anticipative time stages. This analysis suggests that an event tree with 2 branches per node and 7 uncertain stages (resulting in 128 scenarios) will be sufficient to ensure consistent results in EVPI and ECIU at a given discount rate.

### **Acknowledgments**

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